A novel effect of scattered-light interference in misted mirrors

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Abstract

Interference rings can be observed in mirrors clouded by condensation, even in diffuse lighting. The effect depends on individual droplets acting as point sources by refracting light into the mirror, so producing coherent wave-trains which are reflected and then scattered again by diffraction round the same source droplet. The secondary wave-train is weak but still capable of modulating the primary to give a circular interference pattern centred on the image of the observer's eye, regardless of the direction of the incident light.

Initial observations

This study began by chance in my bathroom, looking in a mirror misted up with condensation. I had my back to the light and saw my head simply as a featureless greyish silhouette. The curious thing was that where my eyes would be, I saw two sets of concentric rings staring back at me. They followed the position of my eyes as I moved and if I shut one eye the demon winked back! This psychedelic effect arises because each eye sees its own pattern, centred on the point where its own image should be. A camera sees a single set of rings, centred on the reflection of the lens.

Figure 1 shows a photograph obtained with the mirror placed in direct sunlight, incident at about 20°, though the rings are equally obvious with diffuse lighting. Taking the intensity values from individual pixels in the photograph, the brightness varies by only ±5% from the mean value, though the contrast seems greater to the eye. The rings look smaller as one approaches the mirror, the angular radius of the first bright ring remaining constant at 0.6°. (This value depends on the thickness of the glass, which was 6 mm.) Using a large hand torch as a directional light source, the rings could be seen for angles of incidence from 10° up to about 40° and appeared unchanged over this range. For larger angles the scattered light gets much weaker.

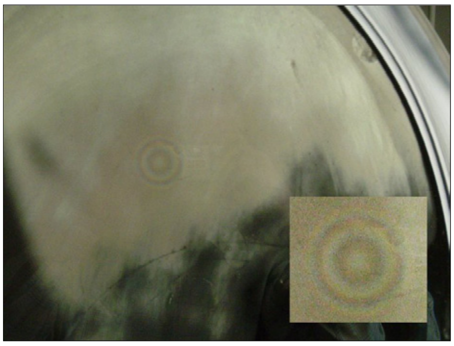

Figure 1. The ring pattern, seen in a misted mirror placed in direct sunlight. The insert is enlarged with enhanced contrast.

The pattern does not appear unless the glass is really clean, so that the condensation forms uniformly. Neither can it be seen if one simply breathes on the glass to make it misty. The condensation process has to be relatively slow and the deposit of droplets relatively light. Looking at the droplets of water on the mirror, using a ×10 hand-lens, I saw an array of circular drops, each with a tiny bright spot where light from the window was focused to form a minute image. I rigged

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Figure 2. Mist droplets on a mirror, lit from the top right. The circle has radius 0.1 mm.

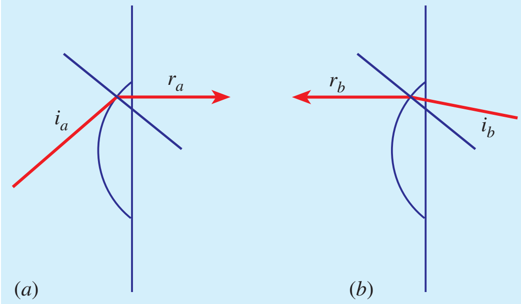

Figure 3. Light rays (*a*) entering and (*b*) leaving through a droplet, in each case being refracted normal to the plane surface.

up a system that allowed me to photograph the drops through a microscope; the results are seen in figure 2. The same area produced good clear rings. There is quite a wide range of drop size but the larger ones are around 30 μm across. The drops are all circular, like tiny lenses, and they are clearly separated. Judging by the profile of large single drops, the contact angle is about $45° \pm 5°$. By contrast, the droplets formed by breathing lightly on the glass are smaller and more closely spaced; further condensation causes them to run together, producing irregular shapes that cover almost all of the surface. Heavy condensation like this never produces the rings. On the other hand, provided the drops are well spaced, drop size seems less important. Sometimes you get a lightly misted area grading into one that scatters strongly, and then by scanning across it is easy to see the effect of a reduction in drop size; all that happens is that the rings get fainter while remaining the same size.

Refraction at the surface of a drop

Figure 2 shows that the cloudy light of the misted surface is due to scattering by refraction. A light ray entering through a drop can be refracted and then reflected back towards the observer, that is along a line perpendicular to the surface of the mirror (figure 3(*a*)), provided the incident ray is within a certain angle of this line. We can apply Snell's law ($\sin i = \mu \sin r$) to estimate the angle; diffraction will introduce only a small uncertainty into the result. The limiting case occurs at grazing incidence so that $i_a = 90°$ and $r_a = 49°$, taking the refractive index of water as 1.33. This shows that the direction of the incident light can be at most $i_a - r_a = 41°$ from the mirror normal if the refracted ray is to be perpendicular to the mirror.

Alternatively the light may be reflected first and then be refracted on exit through the curved surface of a drop, which is what is seen in figure 2. For the refracted ray to be perpendicular to the mirror (figure 3(*b*)), the incident light must be much closer to this line because r_b is limited by the contact angle; taking this as 45° we get $i_b = 32°$ so the incident ray, inside the water, makes an angle of 13° with the mirror normal if the refracted ray is perpendicular to the mirror. Allowing for refraction at the water–glass and air–glass interfaces, the incoming ray must be within 17° of the mirror normal.

In practice these limits will be blurred by the effects of diffraction but there is still a significant difference between the two cases. Since the circle pattern is clearly seen for angles of incidence in the range 20°–40°, we have good evidence that light entering via refraction at a drop is responsible. Furthermore, by restricting the angle of incidence to this range one can ensure that only light entering through a drop contributes to scattering towards the observer, which gives a useful increase in contrast in the ring pattern.

Further experiments: all done with mirrors

A more convenient and reproducible way of forming the mists is to lay a sheet of glass (or a mirror) over a plastic box containing water that is slightly above room temperature: a difference of 3 °C gives the required slow condensation. Watching through the glass, the importance of careful cleaning quickly becomes apparent; glass that looks clean when dry may develop a very patchy appearance as the mist forms. The surface must be free from any deposit of microscopic particles that may nucleate formation of droplets, so that deposition occurs randomly.

Using the water bath technique I was able to reproduce the ring pattern with a smaller glass mirror and also, very faintly, with a sheet of clear glass, provided it was viewed from the misted side. As in the original observations, it was necessary to have my back to the light. Under the same conditions, however, the pattern was not detectable using a sheet of dark blue glass (cobalt glass, or welder's glass) or with a front-silvered mirror. This shows the layer of mist droplets on its own is not sufficient: it must be paired with a mirror image layer, formed by reflection at the back surface. (Clear glass gives about 4% reflection, just enough for the ring pattern to be detectable.)

The scattering seen in a misted mirror resembles that obtained with a thick sheet of glass covered with condensation on both sides, except that in this case the two layers of drops are placed randomly rather than one being a mirror image of the other. It seemed worthwhile to find out if the ring pattern could also be seen with two layers of mist. The most convenient way of doing this is to use two sheets of glass, misted up using the water-bath method. Then each sheet can be separately tested with a mirror to check that it does produce the rings before placing them back to back. To get comparable conditions for a fair test, the observer should look directly through the double layer of glass towards a black background with light incident on the glass from behind at about 30° to the line of sight. No rings were seen when this was tried so one must conclude that a mirror image of the mist is required, in which each drop is opposite its corresponding image.

Placing a misted sheet of glass in front of a clear mirror, one sees the same ring pattern as with a misted mirror, both when the condensation faces outwards and also when it is toward the mirror. The rings are centred on the reflection of the observer's eye in the mirror and tilting the glass has no effect. With this arrangement it is possible to vary the spacing and it immediately became obvious that the rings become smaller as the misted surface is moved away from the mirror, while extra rings become visible on the outside. I was able to get up to four before the pattern became too faint to see. At this stage, with a total of 7 mm of glass and a 7 mm air gap, the first ring had shrunk to about 0.4°. On the other hand, with a 4 mm thick mirror only the first ring is visible, having an angular radius of 0.7°. This variation strongly suggests interference between the light scattered from a drop and from its mirror image, i.e. scattered and reflected.

Scattered-light interference

Interference due to light scattered at the surface of a mirror is well known [1]. It can be seen with a variety of scattering particles, including talc and dew; a recent account [2] describes very colourful fringes due to dust particles sitting on a water surface. It is caused by interference between two rays of light, one being first scattered at the surface and then reflected and the other reflected first before being scattered by the same particle. In all cases a directional source of light is required, since the waves must be coherent if they are to interfere.

I can see interference fringes of this type in the misted mirror by using a small pencil flashlight, held close to my head and pointed at the mirror. The image of the light is surrounded by a corona caused by diffraction of the light by individual drops [3]. The interference pattern is only visible in the central white region of the corona, as parallel coloured bands on either side of the zero-order fringe, which runs directly across the image of the bulb. The spacing of the fringes is very sensitive to the displacement of the light from my eye. Since my little flashlight can be focused to produce a parallel beam, I can actually get all three effects at once by directing the beam onto the mist exactly in front of the reflection of my eye: the corona and the fringes centred on the image of the light and the ring pattern centred on the image of my eye. Of the three the corona is immediately obvious while the rings are the most faint and easily overlooked. Since the fringes only appear inside the corona, they also must arise from light that is scattered by diffraction. The ring pattern is the same angular size so it seems likely also to involve diffraction.

Scattering by refraction and by diffraction

The most striking difference between normal scattered-light interference and the rings described here is that the rings can be seen with diffuse lighting; somehow the need for an external coherent source of light has been removed. Figure 2 suggests a possible explanation: each drop behaves as a pinhole source. Taking the wavelength of visible light to be 0.5 μm, an aperture of diameter 30 μm imposes spatial coherence on waves diverging by up to about 2°. (The light from a drop will spread over a much wider angle than this but more widely divergent waves will not be coherent and so cannot give rise to interference effects.)

Following reflection at the back surface, the coherent wavefront from a 30 μm drop will spread across a circular patch 0.2 mm in diameter on the surface of a mirror 6 mm thick, as shown in figure 2. Any single drop will intercept something of the order of 1% of this light, but this time it is diffraction, not refraction, that is important. The lensing action of the drop will spread the incident light over a wide angle (40° as opposed to 1°) and so reduce its intensity so much as to make any interference undetectable. Diffraction of the light passing by the edge of a drop is much more moderate, spreading it into a conical wavelet with significant intensity since it diverges by just 1°, exactly the same as for a pinhole of the same diameter.

All the wavelets from the droplets in the circle will be coherent with each other and with the incident wavefront (which is much stronger), so we should include them all in calculating the resulting interference pattern for a single point source. Such a pattern can be realized by using a manufactured pinhole placed a few millimetres away from a misted sheet of glass, using either white light or a laser beam focused onto the pinhole to act as source. Since the illuminated area of mist is small, the diffraction pattern can be projected onto a screen, though very faintly. A better idea is to place a camera (**not** your eye!) with its lens close to the spot of light and set the focus to infinity so that the pattern is recorded directly on the light-sensitive surface. It will not look like a simple pattern of concentric rings. So how does this help us explain the observed pattern?

Each droplet in the misted mirror acts as a source and produces its own diffraction pattern. Since the phase relationships between them are random, we can calculate their combined effect simply by adding their intensities, with the result that the observed pattern represents an average of many individual contributions, all different. In most directions the averaging gives a uniform brightness but the forward direction is special because in every case one drop is at the centre of the circle of coherent light from the source, which is of course its own mirror image. The interference patterns of these pairs all line up because the pairs are all aligned at right angles to the surface of the mirror. What survives the averaging process is therefore mainly due to the interference between the light scattered from these two mirror-image centres. Secondary scattering from the nearest neighbours is not totally random and so must also make some contribution but it is not necessary to account for the basic picture.

The interference pattern from a droplet-image pair

Figure 4 compares normal scattering and the double scattering mechanism. (I have 'unfolded' the reflections by drawing all the reflected rays on the left side of the mirror, so that the drop shown at R is actually the image of the real drop at P.) In normal scattering (figure 4(*a*)) parallel light from a distant point source is scattered at P and R, giving rise to two waves of equal intensity that can reinforce or cancel. Double scattering arises (figure 4(*b*)) when light incident on the drop P is scattered towards the mirror, reflected and then scattered again at R to give a second weaker wave, which can only modulate the brightness of the wave from P to give a relatively faint pattern.

Writing the angles of the incident and scattered rays (in the glass, not in air) as $\theta_1 = \angle ORP$ and $\phi_1 = \angle QPR$, then for normal scattering the path difference

$$d = \overline{OR} - \overline{PQ} = 2t(\cos(\theta_1) - \cos(\phi_1)) \simeq t(\phi_1^2 - \theta_1^2) \qquad (1)$$

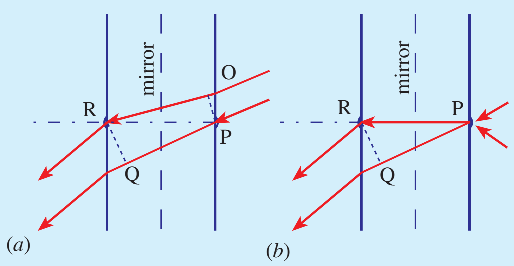

Figure 4. Comparison of (*a*) normal scattered-light interference and (*b*) interference due to secondary scattering at R.

where t is the thickness of the glass and the approximation is good for small angles. Bright fringes occur when d is a whole number of wavelengths, $n\lambda_1$ (again, as measured in the glass); zero-order implies $\theta_1 = \phi_1$. By contrast, for the double scattering

$$d = \overline{PR} - \overline{PQ} = 2t(1 - \cos(\phi_1)) \simeq t\phi_1^2 \quad (2)$$

and since the dependence on θ_1 is lost, the zero-order corresponds to $\phi_1 = 0$ and the resulting pattern is circular, centred about the normal. For the nth ring, $d = n\lambda_1 = n\lambda_0/\mu$ and (for small angles) $\phi_1 = \phi_0/\mu$ where ϕ_0 and λ_0 are the values in air and μ is the refractive index of the glass. Hence

$$\phi_0 = \sqrt{n\lambda_0\mu/t}. \quad (3)$$

(For a misted glass held in front of a mirror, t is calculated by adding the air gap *multiplied* by μ to the total thickness of glass.) This formula reproduces the observed angular sizes quite well for the first-order ring but precise measurements would require the use of monochromatic light.

The effects of drop size

The drops will obviously scatter more light as they increase in size, so that the grey haze of the mist becomes more marked. In the same way, the proportion of the light that is scattered twice must also increase, so the ring pattern will become more easily visible. In addition, diffraction decreases for larger drops, so not only is more light scattered a second time but that light is concentrated into a smaller part of your field of view. The result is that the contrast in the interference pattern increases sharply with drop size, so that the ring pattern is not visible in a newly formed mist. The drops have to continue growing until they get close to 20 μm diameter. Consequently other materials like talc, which are often used to demonstrate scattered-light interference, do not show the double-scattering effect at all because the particles are too small.

We can estimate the contrast in the ring pattern by comparing the diffraction with the angle α subtended by the drop at its effective source. For small angles, $\alpha = D/2t$, where D is the diameter of the drop and t the thickness of the glass. For a drop this size, the angular diameter of the coherent wavefront produced by diffraction is given by $\beta = 2\lambda/D$. The second scattering produces a cone of light that is expanded in the ratio β/α, so the intensity is reduced by a factor $(\alpha/\beta)^2$. However, the contrast in the interference pattern is determined by the field vector, which is only reduced by the factor

$$\alpha/\beta = D^2/4t\lambda. \quad (4)$$

The observed variation in intensity of the ring pattern of $\pm 5\%$ implies that the field strength of the doubly scattered wave is 5% of the singly scattered wave and hence $D \simeq 25$ μm.

However, the outer limit of the ring pattern is also determined by diffraction; if the angle β becomes less than 0.5°, we will not be able to see even the first bright ring. The large drop in figure 2 is already past this limit. It was just a matter of chance that the conditions in my bathroom happened to favour formation of mists with the optimum drop size!

A mirror can easily be covered with a mist of large droplets from a fine spray. The drops still scatter light by refraction and from a distance it is hard to tell the difference. However, light rays refracted exactly perpendicular to the mirror will be blocked since they are reflected back to the drop from which they came and the lensing effect spreads the light over a wide angle. This means that each droplet casts a shadow of itself in the forward direction. It does not follow that a shadow will be seen on a screen held opposite the mirror, of course, since light is scattered from the whole misted area. However, if you look straight at the mirror with the light behind you, you see your silhouette as before but this time your eyes become areas of deeper shadow. The demon has become a ghoul! This happens because the drops immediately opposite your eye block the scattered light that has been reflected back towards you. All round this central shadow there is a circular region where the scattered light has passed through the clear area surrounding each drop, so none is blocked and this region is actually slightly brighter that the average. The effect is analogous to the formation of moiré fringes between identical wire grids; if two such grids are held parallel but with one further back from the observer, an observer sees a central bright area where the holes are aligned, surrounded by a dark region where the holes are not in alignment because of the effect of perspective. However, the moiré pattern repeats at regular intervals whereas the random array of droplets gives just the one dark patch.

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